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DIFFRACTION OF A PLANE ELASTIC WAVE BY A WEDGE WITH A SPECIAL FORM OF BOUNDARY CONDITIONS†

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An exact solution of the antiplane problem of the diffraction of a plane elastic SH-wave with a step profile by a wedge is obtained. The stresses on the wedge sides are assumed to be proportional to a linear combination of the displacements, velocities and higher derivatives with respect to time of the displacements along the wedge axis. A solution of the problem is obtained using integral transformations with subsequent transformation using Cagniard's method. Solutions of the corresponding problems with boundary conditions of the Winkler and inertial types are considered. When a wave with a linear profile is incident on the wedge the stresses suffer a discontinuity of the second kind on the diffraction wave front; the same type of feature is observed in the problem with the inertial condition. © 1999 Elsevier Science Ltd. All rights reserved.

A similar problem with viscous friction on the boundary was considered previously [1].

We will consider the dynamic problem for a wedge-shaped region $r \geqslant 0$, $\alpha \leqslant \theta \leqslant 2\pi - \alpha$, $-\infty < z + \infty$ ($r, \theta z$ are cylindrical coordinates) of the diffraction of a plane SH-wave, the front of which is parallel to the edge of the wedge

$$\Delta v = \frac{\partial^2 v}{\partial t^2}, \quad r > 0, \quad \alpha \leqslant \theta \leqslant 2\pi - \alpha$$

$$\pm \frac{1}{r}\frac{\partial v}{\partial r} = \sum_{m=0}^{n} k_m \frac{\partial^m v}{\partial t^m}, \quad \theta = \pm\alpha \tag{1}$$

$$v = C(t) + o(r^\lambda), \quad r \to 0; \quad \lambda > 0, \quad v = v_0 = H(t - \cos(\theta - \theta_0)), \quad t < 0$$

where $H(t)$ is the Heaviside function. It can be shown by direct substitution that the solution of this system can be sought in the form

$$v = \frac{1}{2\pi i}\int_L \left(\int_{c-i\infty}^{c+i\infty} A(p,s) b(p,s) \exp(p(t - r\varphi(s,\theta))) dp \right) ds \tag{2}$$

$$b(p,s) = \frac{p^{n-1}\psi(s)}{P_n(p,s)}, \quad P_n(p,s) = \sum_{m=0}^{n} k_m p^m + p\psi(s),$$

$$\varphi(s,\theta) = s\cos\theta + \sqrt{1-s^2}\sin\theta, \quad \psi(s) = (\sin\alpha)s + \sqrt{1-s^2}\cos\alpha$$

The contour L lies in the region $\operatorname{Re} s > 0$ for $\alpha \leqslant \theta < \pi$, and we correspondingly choose the positive branch of the radical; when it lies in the region $\pi \leqslant \theta \leqslant 2\pi - \alpha$ we choose the negative branch. Expression (2) when $b(p, s) \equiv p^{-1}$ should be the solution of the problem of the diffraction of a plane SH-wave by a smooth rigid wedge.

We expand the fraction $b(p, s)$ into simple fractions

$$b(p,s) = \sum_l \sum_{j=1}^{n_l} \frac{g_{lj}(s)}{(p-p_l)^j}$$

where p_1 are the roots of the polynomial $P_n(p, s)$ and n_1 are their multiplicities. Since the coefficients of this polynomial have no poles at finite points, the functions $p_1(s)$ are regular everywhere, apart from infinity. In exactly the same way the numerator $g_{1j}(s)$, determined from the system of linear equations with non-zero determinant, has no poles, apart from the point $s = \infty$. Hence the poles of the integrand in (2) are defined as whole by the factor $A(p, s)$, the singular points of which lie on the real straight line and define plane reflected waves.

Hence, the structure of the integrand enables us to deform the initial contour L into a Cagniard contour [2], defined by the relations

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$$s=r^{-1}(\tau\cos\theta\pm i\sqrt{\tau^2-r^2}\sin\theta),\ \tau>r;\ s=r^{-1}(\tau\cos\theta\pm\sqrt{r^2-\tau^2}\sin\theta),\ \tau\leqslant r$$

The expression for the displacements after taking an inverse Laplace transformation has the form

$$v=v_0+\frac{1}{\pi}\operatorname{Im}\sum_l\sum_{j=1}^{n_l}\left(\frac{1}{(j-1)!}\int_0^t A'(s)g_{lj}(s)\exp(p_l(s)(t-\tau))(t-\tau)^{j-1}\frac{ds}{d\tau}d\tau\right) \tag{3}$$

The factor $\exp(p_1(s)t)$ is bounded for a negative value of the real part of the roots $p_1(s)$. The Routh–Hurwitz criterion for polynomials with complex coefficients [3] enables us to establish that this condition is satisfied if the roots of the polynomial $k_0 + k_1p + \ldots + k_np^n$ are negative.

The real part of the function $A(s)$ is none other than the perturbed part of the solution of the problem of the diffraction of an acoustic wave by a rigid wedge. A method of determining it was described in detail in [4].

We will consider some special cases.

Suppose we are given a wedge with an absorbing coating, $k_0 \neq 0$. The boundary condition will be as follows:

$$\sigma_{\theta z}=-k_0v,\ \theta=\pm\alpha$$

The solution of the diffraction problem with this boundary condition is obtained by substituting into (2) the values of the coefficients $k_i = 0$ when $i \neq 0$

$$v=v_0+\frac{1}{\pi}\operatorname{Im}\left(\int_0^t A'(s)\exp\left(\frac{(\tau-t)k_0}{\psi(s)}\right)\right)\frac{ds}{d\tau}d\tau$$

We will investigate the form of the perturbed motion in the region of the diffraction wave front when the incident wave has a linear front

$$v_1=\xi H(\xi),\ \xi=t-r\cos(\theta-\theta_0)$$

The solution of this problem is obtained by convoluting (3) with the expression for the incident wave with υ_1. Assuming the quantity $\varepsilon = t/r - 1$ is small, it can be shown that the displacements in the region of the diffraction wave front are proportional to $\varepsilon^{3/2}$. Consequently, the stresses remain continuous on passing through the diffraction wave front, but the derivative of σ_{rz} with respect to the variable r suffers a discontinuity of the second kind.

In a similar way one can obtain a solution of the problem of the diffraction of a plane wave by a wedge with an inertial boundary condition (the stresses are proportional to the acceleration on the wedge surface)

$$v=v_0+\frac{1}{\pi}\operatorname{Im}\int_0^t A'(s)\exp((\tau-t)\psi(s))\frac{ds}{d\tau}d\tau$$

The behaviour of the stresses in the region of the diffraction wave front is qualitatively the same as in the problem for a wedge with an elastic coating.

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